BWXX SEENET-MTP MEETING

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Recent advances in noncommutative field theories and gravity

Marija Dimitrijević Ćirić

University of Belgrade, Faculty of Physics,
Belgrade, Serbia

based on:

colaborations during the 20 years of SEENET-MTP

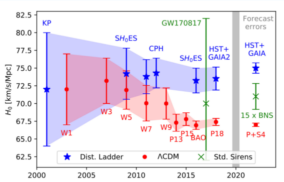
Motivation

What are enementary particles, how do they interact, how did Big Bang happen... Our current theories, Standard Model and ΛCDM give answers to these questions. But not complete answers...

Standard model is not complete: free parameters, GUT, supersymmetry (new particles, dark matter), vacuum energy (cosmological constant, dark energy?). . .

Modification of (quantum) gauge field theories needed.

Problems on quantum and cosmological scales:
-singularities in BH and cosmological solutions, BH evaporation, (so far) no quantum gravity,
-Dark Matter, Dark Energy; more recently: tensions in cosmology



Modifications and quantization of General Relativity needed.

Partial answers to these problems give: string theory, loop quantum gravity, noncommutative geometry...

During the last 20-22 years the HEP group at Faculty of Physics, University of Belgrade has been working on some of these problems, mostly using the approach of noncommutative geometry.

NC historically:
-Heisenberg, 1930: regularization of the divergent electron self-energy, coordinates are promoted to noncommuting operators

$$[\hat{x}^\mu, \hat{x}^\nu] = i\Theta^{\mu\nu} \Rightarrow \Delta\hat{x}^\mu \Delta\hat{x}^\nu \geq \frac{1}{2}\Theta^{\mu\nu}.$$

-First model of a NC space-time [Snyder '47].

More recently:
-mathematics: Gelfand-Naimark theorems (C^*-algebras of functions encodes information on topological Hausdorff spaces),
-string theory (open string in a constant B-field),
-quantum gravity = discretisation of space-time?

Different approaches and different models of NC theories: NC spectral geometry, fuzzy spaces, matrix models, $\star$-product representations...

Overview

Motivation

NC $\star$-gauge theories

Twist deformed gravity and differential geometry

Braided L_∞-algebras and braided QFT

Outlook

NC $\star$-gauge theories

Star-product approach:

$$\begin{array}{ccc} \hat{\mathcal{A}}_{\hat{x}} & \mapsto & \mathcal{A}^{\star}_{x} \\ \hat{f}(\hat{x}) \mapsto f(x) & \text{and} & \hat{f}(\hat{x})\hat{g}(\hat{x}) \mapsto f \star g(x). \end{array}$$

An example: Moyal-Weyl $\star$-product

$$\begin{aligned} f \star g\,(x) &= \sum_{n=0}^{\infty} \left(\frac{i}{2}\right)^n \frac{1}{n!} \theta^{\rho_1\sigma_1} \ldots \theta^{\rho_n\sigma_n} \Big(\partial_{\rho_1} \ldots \partial_{\rho_n} f(x)\Big) \Big(\partial_{\sigma_1} \ldots \partial_{\sigma_n} g(x)\Big) \\ &= f \cdot g + \frac{i}{2}\theta^{\rho\sigma}(\partial_\rho f) \cdot (\partial_\sigma g) + \mathcal{O}(\theta^2)\,, \\ & \quad [x^\mu \stackrel{\star}{,} x^\nu] = i\theta^{\mu\nu}\,. \end{aligned}$$

Associative, noncommutative; c. conjugation: $(f \star g)^* = g^* \star f^*$. Derivatives, integral are well defined

$$\int \mathrm{d}^4x\ f \star g = \int \mathrm{d}^4x\ g \star f = \int \mathrm{d}^4x\ fg.$$

$\star$-product can be obtained via: deformation quantization [Bayen et al. '78], formality map [Kontsevich '98], "twist" formalism [Aschieri et al. '06, '08]...

Field theories on κ-Minkowski space-time

κ-Minkowski: NC space-time with quantum symmetry, κ Poincaré symmetry:

$$[x^0 \stackrel{\star}{,} x^j] = i\frac{1}{\kappa}x^j, \quad [x^j \stackrel{\star}{,} x^k] = 0.$$

Deformed dispersion relations for sclar and spinor fields, $U(1)_\star$ gauge theory [MD... Wess '03;... MD, Jonke, Pachol '14].

BW03

BSI 2011

NC $SO(2,3)_\star$ gravity

$SO(2,3)_\star$ NC gravity model is based on:

-NC space-time: Moyal-Weyl NC space-time with the small deformation parameter $\theta^{\alpha\beta} = -\theta^{\beta\alpha}$ and the Moyal-Weyl $\star$-product..

-gravity: $SO(2,3)_\star$ gauge theory with symmetry broken down to $SO(1,3)$, [Stelle, West '80; Wilczek '98] and [MDC, Nikolić, Radovanović '17;...MDC, Gocanin, Djordjevic, Nikolić, Radovanović '23]

-Seibrag-Witten map: connects NC fields and the corresponding commutative fields.

Some results:

NC gravity action is a perturbative expansion in $\theta^{\alpha\beta}$:

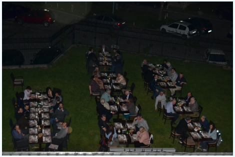
$$S_{NC}^{(0)} = \text{ GR }, \quad S_{NC}^{(1)} = 0, \quad S_{NC}^{(2)} \neq 0.$$

NC corrections to Minkowski space-time

$$g_{00} = 1 - R_{0m0n}x^m x^n, \quad g_{0i} = 0 - \frac{2}{3}R_{0min}x^m x^n, \quad g_{ij} = -\delta_{ij} - \frac{1}{3}R_{imjn}x^m x^n$$

Breaking of the diffeomorphism symmetry is a choice of a prefered reference frame! Only observers in Fermi normal coordinates see constant noncommutativity.

We started talking about this NC gravity model around the time of BW2013 in Vrnjacka Banja 2013.



Twist deformed gravity and differential geometry

Start from a symmetry algebra g and its universal covering algebra Ug. Then define a twist operator $\mathcal{F}$ as:

-an invertible element of $Ug \otimes Ug$
-fulfills the 2-cocycle condition (ensures the associativity of the $\star$-product).

$$\mathcal{F} \otimes 1(\Delta \otimes \mathrm{id})\mathcal{F} = 1 \otimes \mathcal{F}(\mathrm{id} \otimes \Delta)\mathcal{F}.$$

-additionaly: $\mathcal{F} = 1 \otimes 1 + \mathcal{O}(h)$; h-deformation parameter.

Braiding (noncommutativity): controled by the R-matrix $\mathcal{R} = \mathcal{F}^{-2} = \mathrm{R}^k \otimes \mathrm{R}_k$; triangular $\mathcal{R}_{21} = \mathcal{R}^{-1} = \mathrm{R}_k \otimes \mathrm{R}^k$.

$$\text{Symmetry Hopf algebra } Ug \quad \xrightarrow{\mathcal{F}} \quad \text{Twisted symmetry Hopf algebra } Ug^{\mathcal{F}}$$
$$\text{Module algebra } \mathcal{A} \quad \xrightarrow{\mathcal{F}} \quad \star \text{ module algebra } \mathcal{A}_\star$$
$$a, b \in \mathcal{A},\ a \cdot b \in \mathcal{A} \quad \xrightarrow{\mathcal{F}} \quad a \star b = \cdot \circ \mathcal{F}^{-1}(a \otimes b) = \mathrm{R}_k(b) \star \mathrm{R}^k(a).$$

Well known example: Moyal-Weyl twist $\mathcal{F} = e^{-\frac{i}{2}\theta^{\rho\sigma}\partial_\rho \otimes \partial_\sigma}$

$$\begin{aligned} f \star g\,(x) &= \cdot \circ \mathcal{F}^{-1}(f \otimes g) \\ &= f \cdot g + \frac{i}{2}\theta^{\rho\sigma}(\partial_\rho f) \cdot (\partial_\sigma g) + \mathcal{O}(\theta^2) = \mathrm{R}_k g \star \mathrm{R}^k f \neq g \star f. \end{aligned}$$

Associative, noncommutative: $\mathcal{R}^{-1} = \mathrm{R}_k \otimes \mathrm{R}^k$ encodes the noncommutativity.

NC conservation laws

Abelian twist that violates translational invariance

$$\mathcal{F} = e^{-\frac{ia}{2}(\partial_t \otimes \partial_\varphi - \partial_\varphi \otimes \partial_t)},$$

with a small deformation paramer a. $\star$-product:

$$f \star g = \mu_\star(f \otimes g) = \mu \mathcal{F}^{-1}(f \otimes g) = fg + \frac{ia}{2}(\partial_t f(\partial_\varphi g) - \partial_t g(\partial_\varphi f)) + \mathcal{O}(a^2).$$

In Cartesian coordinates NC is Lie algebra type

$$[t \stackrel{\star}{,} x] = -iay, \quad [t \stackrel{\star}{,} y] = iax. \tag{1}$$

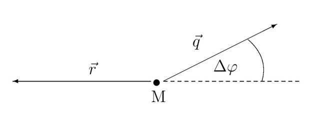
Twisted Poencaré Hopf algebra

$$\Delta^{\mathcal{F}} P_1 = P_1 \otimes \cos\left(\frac{\theta}{2}P_0\right) + \cos\left(\frac{\theta}{2}P_0\right) \otimes P_1 + P_2 \otimes \sin\left(\frac{\theta}{2}P_0\right) - \sin\left(\frac{\theta}{2}P_0\right) \otimes P_2, \tag{2}$$

$$\Delta^{\mathcal{F}} P_2 = P_2 \otimes \cos\left(\frac{\theta}{2}P_0\right) + \cos\left(\frac{\theta}{2}P_0\right) \otimes P_2 - P_1 \otimes \sin\left(\frac{\theta}{2}P_0\right) + \sin\left(\frac{\theta}{2}P_0\right) \otimes P_1,$$

...

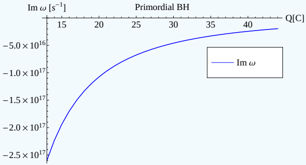
Direct consequence: modified conservation law of momenta [MDC, Konjik, Kurkov, Lizzi, Vitale '18]! For a particle of mass M, decaying from rest

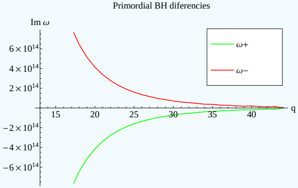


with $\Delta\varphi = \pi - \frac{aM}{2}$. Potential experimental checks?

NC quasinormal modes

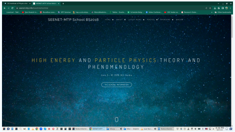
(Breaking of rotational symmetry leads to violation of angular momentum conservation: spliting (m-dependence) of QNM spectrum of a NC scalar field in the charged BH geometry [MDC, Konjik, Samsarov '18...'23]





with $\omega^{\pm} = \omega(m = \pm 1) - \omega(m = 0)$. For a primordial BH with $M = 8.8 \times 10^{12}\ kg$ and $13C \leq Q \leq 44C$ and a massless scalar field with $q = e$ and $l = 1$: $\delta_{\rm Im} \sim \frac{{\rm Im}\,\omega^{+}}{{\rm Im}\,\omega} \sim \frac{10^{14}}{10^{17}} \sim 10^{-3}$.

Experimental check: QNM spectrum of NC gravitational and/or vector field needed.



Braided L_∞-algebras and braided QFT

L_∞-algebra (strong homotopy algebra): generalization of a Lie algebra with higher order brackets that fulfil homotopy relations.

In physics: higher spin gauge theories with field-dependent gauge parameters [Berends, Burgers, van Dam '85]; Generalized gauge symmetries of closed string field theory [Zwiebach '15]; Any classical field theory with generalized gauge symmetries is determined by an L_∞-algebra, duality with BV-BRST [Hohm, Zwiebach '17; Jurčo et al. '18]. Applications to QFT: correlation functions, amplitudes, double copy [Arvanitakis '19; Jurco et al. '20; Borsten et al. '21]...

NC gauge field theories in the L_∞ setting first discussed in [Blumenhagen et al.'18; Kupriyanov '19].

L_∞-algebras of ECP gravity, classical and noncommutative [MDC, Giotopoulos, Radovanović, Szabo '20, '21]; Braided NC (quantum) field theories from the baided L_∞-algebra [MDC, Konjik, Radovanović, Szabo '22, '23].

Braided quantum ϕ^n theory

Braided scalar field theory: 4D Minkowski space-time, Moyal-Weyl twist and a real massive scalar field ϕ with ϕ^n, $n \geqslant 3$ interaction.

Classical theory is given by the graided vector space $V = V_1 \oplus V_2$ with $V_1 = V_2 = \Omega^0(\mathbb{R}^{1,3})$ and

$$\ell_1(\phi) = -(\Box + m^2)\phi, \ \ \ell_{n-1}(\phi_1, \dots, \phi_{n-1}) = -\lambda \phi_1 \cdot \dots \cdot \phi_{n-1},$$
$$\langle \phi, \phi^+ \rangle = \int \mathrm{d}^4x \ \phi \ \phi^+,$$
$$S(\phi) = \frac{1}{2}\langle \phi, \ell_1(\phi), \phi \rangle - \frac{1}{n!}\langle \phi, \ell_{n-1}(\phi, \phi, \phi) \rangle = \int \mathrm{d}^4x \ \Big(\frac{1}{2}\phi \left(-\Box - m^2 \right) \phi - \frac{\lambda}{n!}\phi^n \Big).$$

Braided NC scalar field theory: the same vector space V with

$$\ell_1^\star(\phi) = -(\Box + m^2)\phi, \quad \ell_{n-1}^\star(\phi_1, \dots, \phi_{n-1}) = -\lambda \phi_1 \star \dots \star \phi_{n-1}$$
$$\langle \phi, \phi^+ \rangle_\star = \int \mathrm{d}^4x \ \phi \star \phi^+,$$
$$S_\star(\phi) = \frac{1}{2}\langle \phi, \ell_1(\phi) \rangle_\star - \frac{1}{n!}\langle \phi, \ell_{n-1}^\star(\phi, \dots, \phi) \rangle_\star = \int \mathrm{d}^4x \Big(\frac{1}{2}\phi \left(-\Box - m^2 \right) \phi + \frac{\lambda}{n!}\phi_\star^n \Big).$$

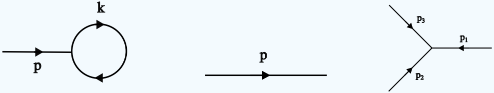
At the classical level, this action is the same as in the usual $\phi_\star^n$ theory! Differences appear at the quantum level. Global symmetries (Lorentz...) discussed in [Giotopoulos, Szabo '22].

BV quantization: braided ϕ^3

Tree level

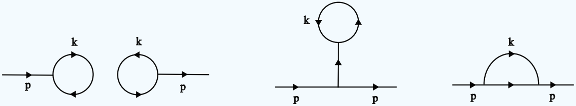
$$G_1^\star(p_1)^{(0)} =\sim \mathrm{i}\,\hbar(-\frac{\lambda}{3!})\frac{\delta(p_1)}{p_1^2-m^2}\int_k \frac{1}{k^2-m^2}\,,\quad G_2^\star(p_1,p_2)^{(0)} = (2\pi)^d\,\frac{\delta(p_1+p_2)}{p_1^2-m^2}\,,$$

$$G_3^\star(p_1,p_2,p_3)^{(0)} \sim (\mathrm{i}\,\hbar)^2(-\frac{\lambda}{3!})\Big(\frac{\delta(p_1)}{p_1^2-m^2}\,\frac{\delta(p_2+p_3)}{p_2^2-m^2}\int_k\frac{1}{k^2-m^2}+\frac{\delta(p_1+p_2+p_3)\,e^{-\frac{\mathrm{i}}{2}\sum_{a<b} p_a\cdot\theta\, p_b}}{(p_1^2-m^2)(p_2^2-m^2)(p_3^2-m^2)}\Big).$$



2-point function at 1-loop:

$$G_2^\star(p_1,p_2)^{(1)} = -\frac{(\hbar\,\lambda)^2}{4}\,\frac{(2\pi)^6\,\delta(p_1)}{p_1^2-m^2}\,\Big[\int_k\frac{1}{k^2-m^2}\Big]^2\,\frac{(2\pi)^6\,\delta(p_2)}{p_2^2-m^2}$$
$$-\frac{(\hbar\,\lambda)^2}{2}\,\frac{(2\pi)^6\,\delta(p_1+p_2)}{(p_1^2-m^2)\,(p_2^2-m^2)}\int_k\frac{1}{k^2-m^2}\Big(\frac{1}{(0-m^2)}+\frac{1}{(p_1-k)^2-m^2}\Big)\,.$$



No NC contributions, no nonplanar diagrams and no UV/IR mixing at 1-loop!

Consistent with [Oeckel '00].

Outlook

- Compare different solutions of $SO(2,3)_\star$ gravity and braided NC gravity.
- Understand better conservation laws in braided NC theories (angular twist, more complicated deformations)
- Look into vector and gravitational QNMs.
- Understand quantization of braided NC field theories, in particular braided Yang-Mills.

- For all this we need at least another 20 years of SEENET-MTP!



HAPPY BIRTHDAY GORAN!!!