TACHYON CONSTANT-ROLL INFLATION IN RSII COSMOLOGY

BWXX SEENET-MTP MEETING

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M. Stojanović
University of Niš, Serbia

In collaboration with:
N. Bilić,
G. S. Djordjević,
D. D. Dimitrijević and
M. Milošević

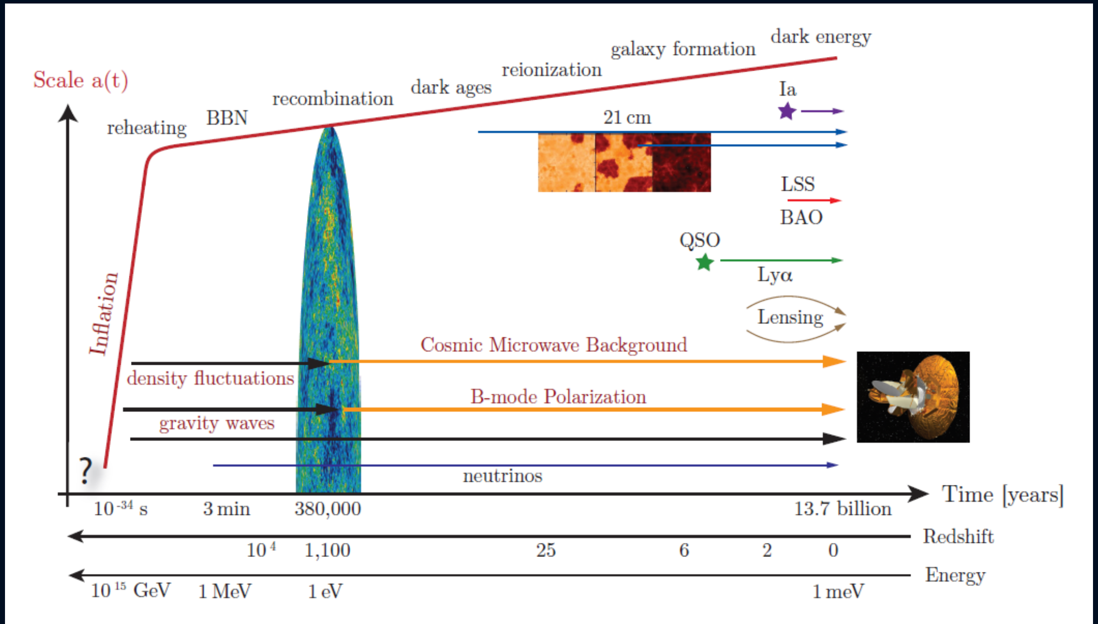
Outline

- Inflation
- Slow-roll parameters
- The constant-roll inflation
- Braneworld cosmology (Randall-Sundrum II model)
- The constant-roll inflation with a tachyon field
- The attractor behavior
- Constant-roll inflation in RSII holographic model
- Conclusions

Inflation

- The ***inflation theory*** proposes a period of extremely rapid (exponential) expansion of the universe during the an early stage of evolution of the universe.

- The inflation theory predicts that during inflation (it takes about $10^{-34}\ s$) radius of the universe increased, at least $e^{60} \approx 10^{26}$ times.



- Although inflationary cosmology has successfully complemented the Standard Model, the process of inflation, in particular its origin, is still largely unknown.

- Recent years brought us a lot of evidence from WMAP and Planck observations of the CMB

- The most important way to ***test inflationary cosmological models*** is to compare the computed and measured values of the ***observational parameters.***

Standard single field inflation

- The Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = c^2dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

- The Friedmann equations

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

- The simplest model of inflation - standard single scalar field inflation $\phi -$ inflaton

$$S = -\frac{1}{16\pi G}\int \sqrt{-g}\, R d^4x + \int \sqrt{-g}\, \mathcal{L} d^4x$$

- Energy density and pressure

$$\rho = \frac{1}{2}\dot{\phi} + V(\phi) \quad p = \frac{1}{2}\dot{\phi} - V(\phi)$$

$$\rho = -3H(\rho + p)$$

- A condition for inflation (from the Friedmann equations)

$$\frac{d}{dt}(aH)^{-1} < 0 \iff \ddot{a} = \frac{d^2a}{dt^2} > 0 \iff \rho + p < 0$$

$a(t)$ - scale factor

$H(t) = \frac{\dot{a}(t)}{a(t)}$ - the Hubble expansion rate

k - the spatial curvature parameter

Slow-roll parameters

- The slow-roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2} \qquad \eta = -\frac{\ddot{H}}{2H\dot{H}} \qquad \frac{\ddot{a}}{a} = H^2(1-\epsilon), \quad \epsilon < 1 \Rightarrow \ddot{a} > 0$$

- The horizon-flow parameters

$$\varepsilon_0 \equiv H_* / H, \qquad \varepsilon_{i+1} \equiv \frac{d \ln|\varepsilon_i|}{dN}, \quad i \geq 0 \qquad \dot{\varepsilon}_i = H\varepsilon_i\varepsilon_{i+1} \qquad N = \int H dt$$

- Example: canonical scalar field

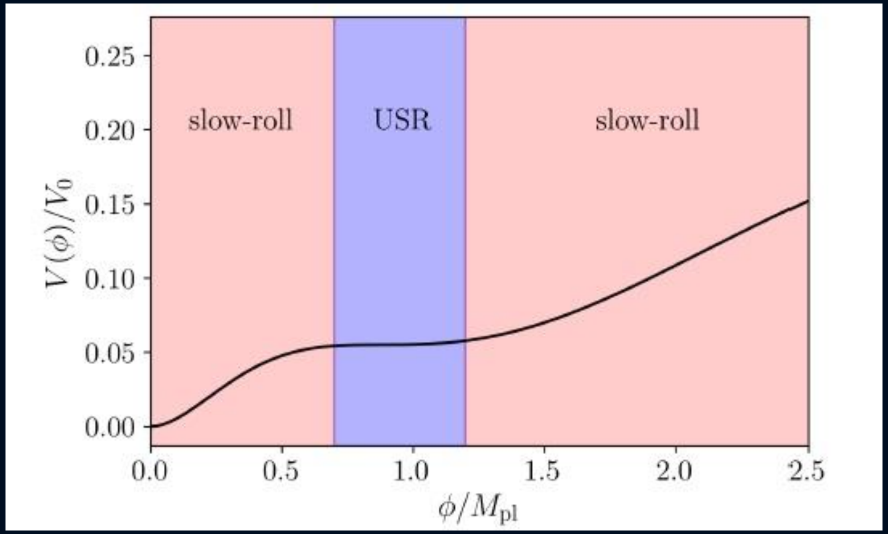
$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}}$$

$$\epsilon \equiv \varepsilon_1$$

$$\eta = \varepsilon_1 - \frac{1}{2}\varepsilon_2$$

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0, \qquad V' = \frac{dV}{d\phi}$$

PBH



- **SLOW-ROLL INFLATION**

$$|\ddot{\phi}| \ll |3H\dot{\phi}| \qquad \ddot{\phi} \approx 0 \quad 3H\dot{\phi} \simeq V'$$

$$|\ddot{\phi}| \ll |V'| \qquad \epsilon \ll 1 \quad |\eta| \ll 1$$

- **CONSTANT-ROLL INFLATION**

$$V' \approx 0 \quad \ddot{\phi} + 3H\dot{\phi} \simeq 0 \qquad \eta = \text{const}$$

$$V' = 0 \quad \Rightarrow \quad \eta = 3$$

ULTRA-SLOW ROLL

The constant-roll inflation

$$\left.\begin{aligned}\eta &= \varepsilon_1 - \frac{1}{2}\varepsilon_2\\ \eta &= \text{const.}\end{aligned}\right\}$$

$$\ddot{H} + 2\eta H\dot{H} = 0$$

- Trivial solution

$$H(t) = \frac{1}{\eta t + c} \qquad \varepsilon_1 = \text{const} \qquad \varepsilon_2 = 0$$

- Nontrivial solutions

$$H_1(t) = -\frac{\beta}{\eta}\tan(\beta t + \gamma)$$

$$\varepsilon_1(t) = \frac{\eta}{\sin^2(\beta t + \gamma)}$$

$$\varepsilon_2(t) = 2\eta\cot^2(\beta t + \gamma)$$

$$N(t) = \frac{1}{\eta}\log\cos(\beta t + \gamma) + C$$

$\eta > 0$

$$H_2(t) = \frac{\beta}{\eta}\cot(\beta t + \gamma)$$

$$\varepsilon_1(t) = \frac{\eta}{\cos^2(\beta t + \gamma)}$$

$$\varepsilon_2(t) = 2\eta\tan^2(\beta t + \gamma)$$

$$N(t) = \frac{1}{\eta}\log\sin(\beta t + \gamma) + C$$

$\eta > 0$

$$H_3(t) = \frac{\beta}{\eta}\tanh(\beta t + \gamma)$$

$$\varepsilon_1(t) = -\frac{\eta}{\sinh^2(\beta t + \gamma)}$$

$$\varepsilon_2(t) = -2\eta\coth^2(\beta t + \gamma)$$

$$N(t) = \frac{1}{\eta}\log\cosh(\beta t + \gamma) + C$$

$\eta < 0$

The solutions which provide a consistent inflationary model.

$$H_4(t) = \frac{\beta}{\eta}\coth(\beta t + \gamma)$$

$$\varepsilon_1(t) = \frac{\eta}{\cosh^2(\beta t + \gamma)}$$

$$\varepsilon_2(t) = -2\eta\tanh^2(\beta t + \gamma)$$

The parameters ε_i cannot be simultaneously positive, the inflation stage never ends!

The constant-roll inflation

- All solutions H lead to the same function $\varepsilon_1(N)$ and $\varepsilon_2(N)$.

$$\varepsilon_1(N) = \frac{\eta}{1-(1-\eta)e^{2\eta(N-N_f)}} \qquad \varepsilon_2(N) = \frac{2\eta(1-\eta)e^{2\eta(N-N_f)}}{1-(1-\eta)e^{2\eta(N-N_f)}}$$

- The observational parameters

$$n_s \simeq 1-2\varepsilon_{1i}-\varepsilon_{2i} \qquad \varepsilon_{1i}=\varepsilon_1(t=t_i) \quad \varepsilon_{2i}=\varepsilon_1(t=t_i)$$

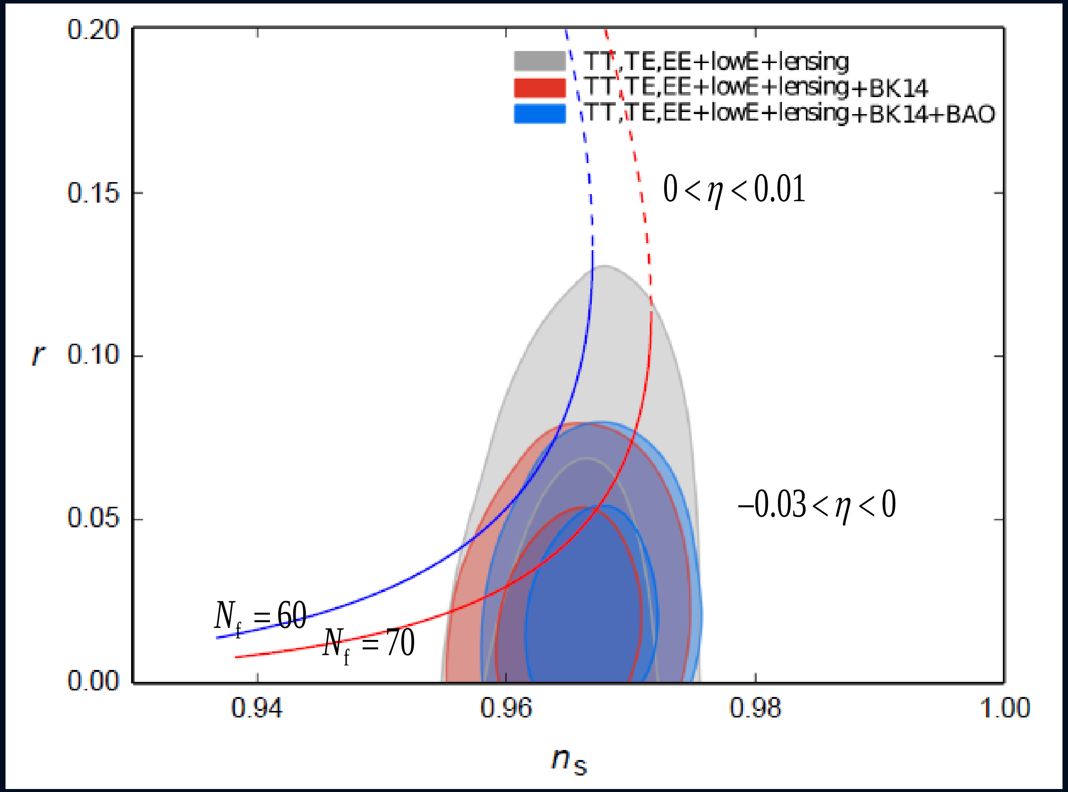
$$r \simeq 16\varepsilon_{1i}$$

- The observational constraints from Planck 2018

$$n_s = 0.9649 \pm 0.0042$$

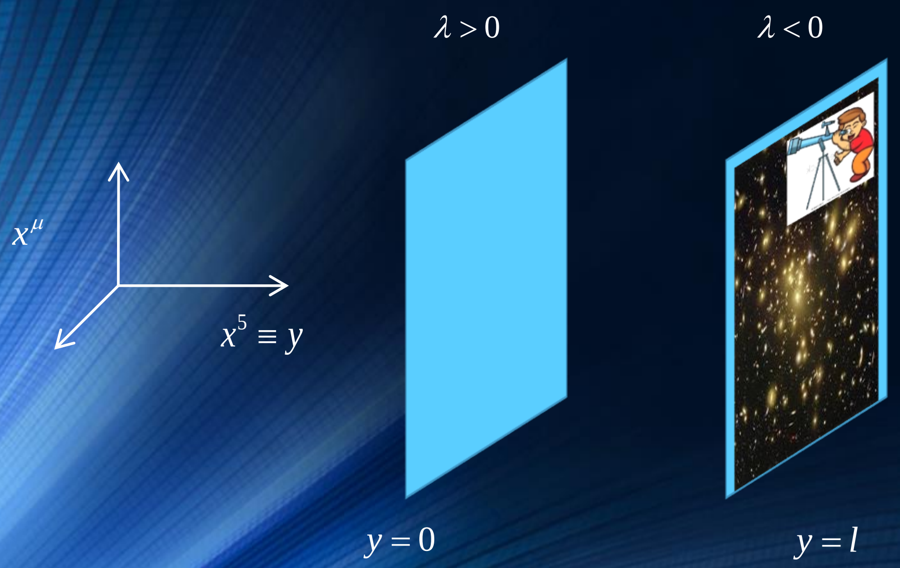
$$r < 0.056$$

- The better agreement is achieved for negative and small values of the parameter η.

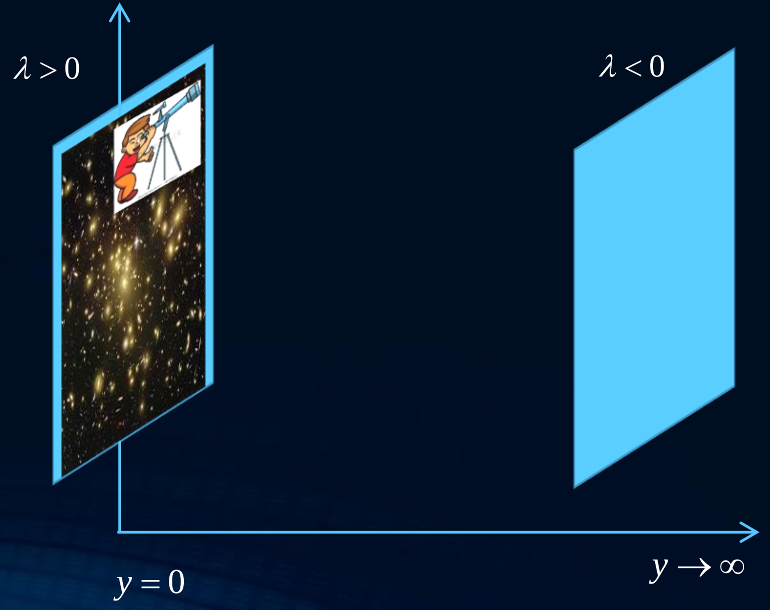


Braneworld cosmology

- Braneworld universe is based on the scenario in which matter is confined on a brane moving in the higher dimensional bulk with only gravity allowed to propagate in the bulk.
- One of the simplest models - Randall-Sundrum (RS) model was originally proposed to solve the hierarchy problem (1999).
- Later it was realized that this model, as well as any similar braneworld model, may have interesting cosmological implications.
- **RS model -** observer reside on the brane with negative tension at $y=l$, distance to the 2nd brane corresponds to the Netwonian gravitational constant.



- **RSII model** – observer is placed on the positive tension brane at $y = 0$, the 2nd brane is pushed to infinity.



Randall-Sundrum II (RSII) model

- The action for the brane world

$$S = \int d^5x\sqrt{-g}\left(\frac{M_5}{2} + \Lambda_5\right) + \int d^4x\sqrt{-h}\left(\mathcal{L} + \lambda\right)$$

the bulk the brane

- Assuming the geometry of the universe to be described by a five-dimensional FLRW metric

$$ds_5^2 = -dt^2 + a^2\delta_{ij}dx^i dx^j + dy^2$$

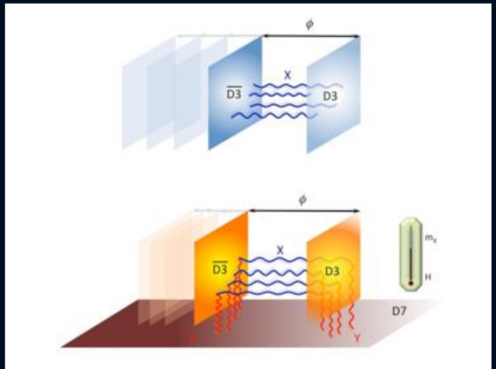
- The usual Friedmann equations are modified so the model has predictions different from the standard cosmology

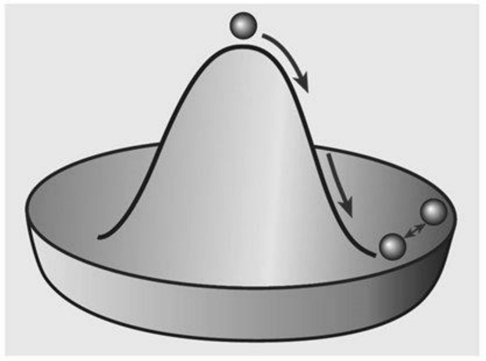
$$H^2 = \frac{8\pi}{3M_4^2}\rho\left(1 + \frac{\rho}{2\lambda}\right) \qquad \lambda = \frac{3}{4\pi}\left(\frac{M_5^3}{M_4}\right)$$

$$\dot{H} = -\frac{4\pi}{M_4^2}(\rho + p)\left(1 + \frac{\rho}{\lambda}\right)$$

Tachyon field

- Traditionally, the word tachyon was used to describe a hypothetical particle which propagates faster than light (Sommerfeld 1904).
- In modern physics this meaning has been changed
  - The effective tachyonic field theory was proposed by A. Sen
  - String theory: states of quantum fields with imaginary mass
    (i.e. negative mass squared)
  - It was believed: such fields permitted propagation faster than light
  - However it was realized that the imaginary mass creates an instability and tachyons spontaneously decay through the process known as tachyon condensation
- No classical interpretation of the ”imaginary mass”
  - The instability: The potential of the tachyonic field is initially at a local maximum rather than a local minimum (like a ball at the top of a hill)
  - A small perturbation - forces the field to roll down towards the local minimum.
  - Quanta are not tachyon any more, but rather an ”ordinary” particle with a positive mass.





Tachyon field

- Tachyon Lagrangian (homogenius and isotropic case)

$$\mathcal{L}(X,\theta) = -V(\theta)\sqrt{1-X}$$

- Tachyon potentia

$$V(0)=\text{const},\quad V_{,\theta}(\theta>0)<0,\quad V(|\theta|\to\infty)\to 0$$

Examples:

$$V(\theta)=\lambda\theta^{-n}$$

$$V(\theta)=\lambda e^{-\theta}$$

$$V(\theta)=\frac{\lambda}{\cosh(\theta/\theta_0)}$$

- The tachyon field can be treated as a fluid with

$$T_{\mu\nu}=(\rho+p)u_\mu u_\nu+g_{\mu\nu},\quad u_\mu=\frac{\partial_\mu\theta}{\sqrt{-\partial_\alpha\theta\partial^\alpha\theta}}$$

$$p=\mathcal{L}=-V\sqrt{1-\dot{\theta}^2}$$

$$\rho=\mathcal{H}=\frac{V}{\sqrt{1-\dot{\theta}^2}}$$

- Friedmann equation

$$H^2=\frac{1}{3M_{Pl}^2}\frac{V}{\sqrt{1-\dot{\theta}^2}}$$

$$\dot{H}=-\frac{1}{2M_{\text{Pl}}}\frac{V\dot{\theta}^2}{\sqrt{1-\dot{\theta}^2}}$$

$$\frac{\ddot{\theta}}{1-\dot{\theta}}+3H\dot{\theta}+\frac{V_{,\theta}}{\theta}=0$$

Canonical field	$\mathcal{L}(X,\phi)=BX-V(\phi)$
Non-canonical models	$\mathcal{L}(X,\phi)=BX^n-V(\phi)$
Dirac-Born-Infeld (DBI) Lagrangian	$\mathcal{L}(X,\phi)=-\frac{1}{f(\phi)}\sqrt{1-2f(\phi)X}-V(\phi)$

$X=\frac{1}{2}\partial_\mu\phi\partial_\nu\phi$ - kinetic energy

$V(\phi)$ - potential

The constant-roll inflation with a tachyon field

- RSII cosmology

$$H^2 = \frac{8\pi}{3M_4^2}\rho\left[1+\frac{\rho}{2\lambda}\right]$$

$$\dot{H} = -\frac{4\pi}{M_4^2}(\rho+p)\left[1+\frac{\rho}{\lambda}\right]$$

$\rho \gg \lambda$

In the high energy regime energy density is larger than the tension of the brane.

$$H^2 \simeq \frac{4\pi}{3M_4^2}\frac{\rho^2}{\lambda}$$

$$\dot{H} \simeq -\frac{4\pi}{M_4^2}\frac{\rho}{\lambda}(\rho+p)$$

Hubble parameter behaves as H ∝ ρ rather than H ∝ √ρ, a novel aspect of the CRI scenario in this context!

- Hamilton-Jacobi formalism $\dot{H} = H_{,\theta}\dot{\theta}$

$$\dot{\theta} = -\frac{n}{3}\frac{H_{,\theta}}{H^2}$$

$n = 1$ RSII cosmology

$n = 2$ Standard cosmology

$$\ddot{H} + 2\eta H\dot{H} = 0 \quad \Rightarrow \quad H_{,\theta\theta}H - H_{,\theta}^2 - 3\frac{\eta}{n}H^4 = 0$$

$$H(\theta) = \frac{2nC_1 e^{\sqrt{C_1}(\theta+C_2)}}{e^{2\sqrt{C_1}(\theta+C_2)} - 3\bar{\eta}C_1}$$

$\bar{\eta} = n\eta$

$C_1 = 1$

$C_2 = 0$

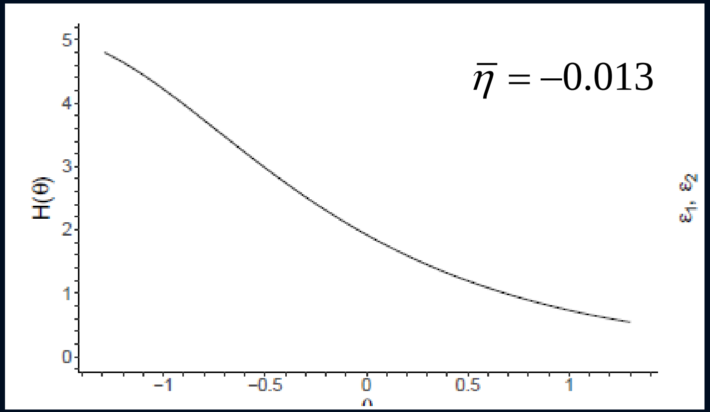
$$H(\theta) = \frac{2ne^{\theta}}{e^{2\theta} - 3\bar{\eta}}$$

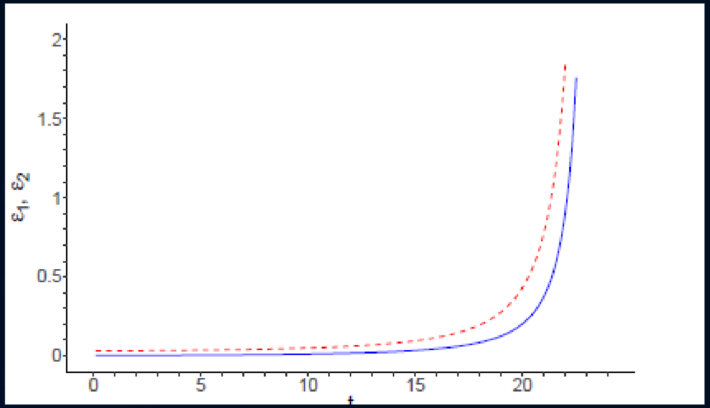
$$H(t) = -\frac{n}{\sqrt{3\bar{\eta}}}\tan(\sqrt{\bar{\eta}/3}\,t + 2C_3), \quad \bar{\eta} > 0$$

$$a(t) \propto \left[\cos\left(\sqrt{\bar{\eta}/3}\,t + 2C_3\right)\right]^{\frac{n}{\bar{\eta}}}, \quad \eta > 0$$

$$H(t) = -\frac{n}{\sqrt{3|\bar{\eta}|}}\tanh(\sqrt{|\bar{\eta}|/3}\,t + 2C_3), \quad \bar{\eta} < 0$$

$$a(t) \propto \left[\cosh\left(\sqrt{|\bar{\eta}|/3}\,t + 2C_3\right)\right]^{-\frac{n}{|\bar{\eta}|}}, \quad \eta < 0$$





The observational parameters

- The inflation parameters in the second order in the slow-roll parameters

$$n_s = 1 - 2\varepsilon_{1i} - \varepsilon_{2i} - \left(2\varepsilon_{1i}^2 + (2C' + 3 - 2\alpha)\varepsilon_{1i}\varepsilon_{2i} + C'\varepsilon_{2i}\varepsilon_{3i}\right)$$

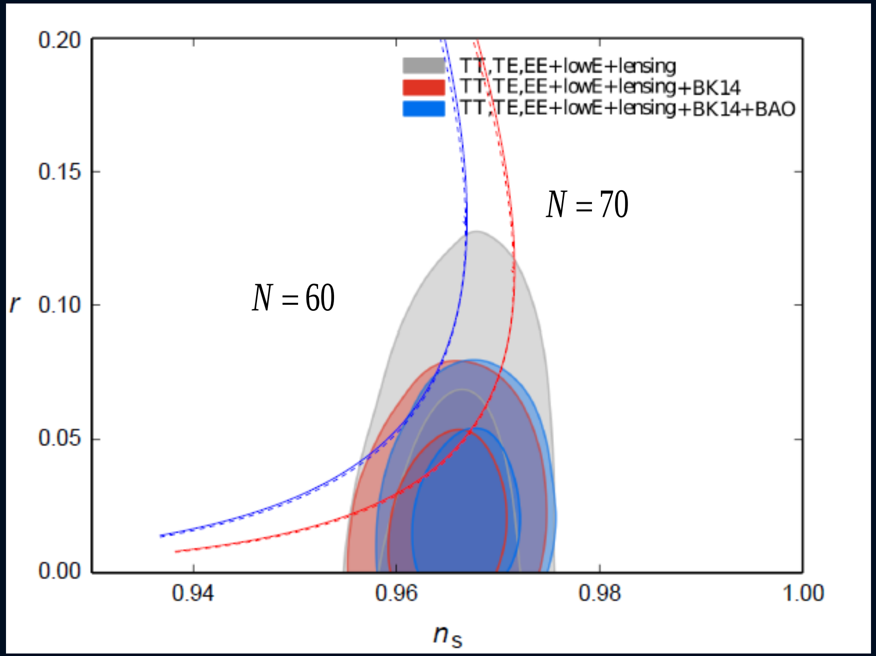
$$r = 16\varepsilon_{1i}\left(1 + C'\varepsilon_{2i} - 2\alpha\varepsilon_{1i}\right)$$

$\alpha = 1/6$ RSII cosmology

$\alpha = 12/6$ Standard cosmology

$C' = -0.72$

$\varepsilon_3 = 2\varepsilon_1$



- A better agreement of analytical and observational results is evident for higher values of *N.*

- The influence of the second order in the slow-roll parameters is insignificant.

The attractor behavior

The attractor behavior of the solution could be investigated:

- analytically: Substituting $H = H_0 + \delta H$

in the Hamilton-Jacobi equation, one could obtain an evolution equation for the perturbation. If the perturbation decays the solution is assumed to be stable.

- numerically: the attractor behavior is considered by plotting the phase space diagram.

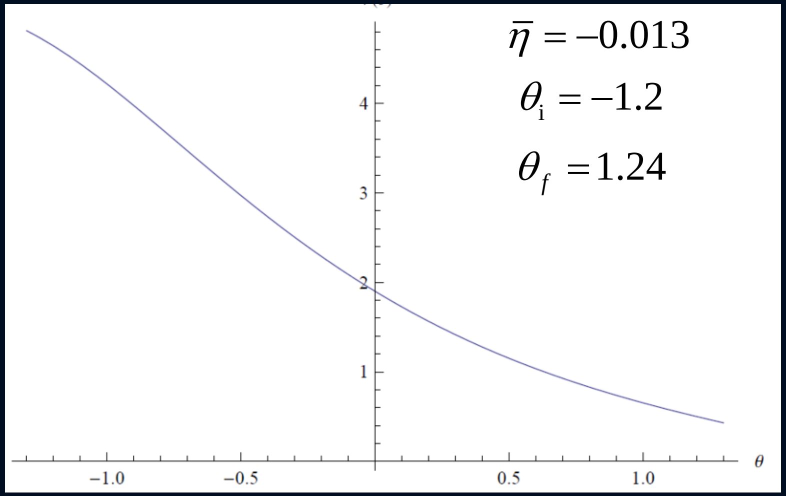
- The reconstructed potentials

$$\rho = V\sqrt{1-\dot{\theta}^2} \qquad H^2 \simeq \frac{4\pi}{3M_4^2}\frac{\rho^2}{\lambda}$$

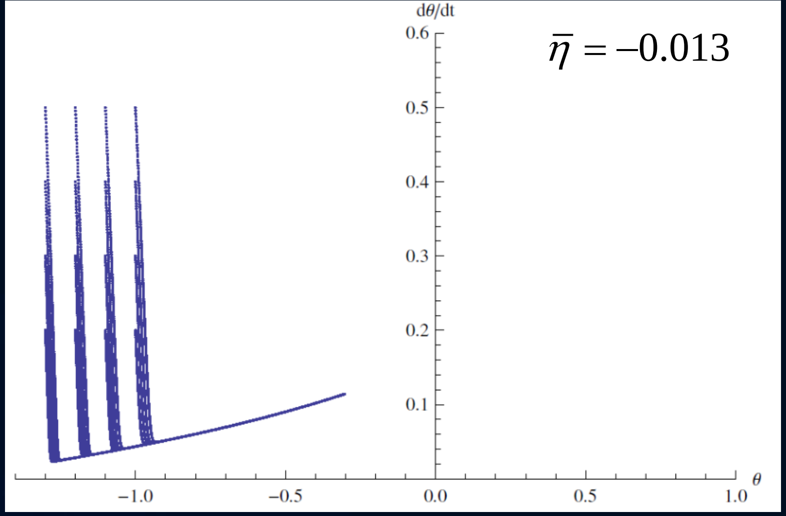
$$\dot{\theta} = -\frac{n}{3}\frac{H_{,\theta}}{H^2} \qquad H = \frac{2ne^{\theta}}{e^{2\theta}-3\bar{\eta}}$$

$$V = \frac{3M_5^3}{4\pi}H\sqrt{1-\frac{1}{9}\frac{H_{,\theta}^2}{H^4}} = \frac{3M_5^3}{4\pi}\sqrt{\frac{36e^{2\theta}-(e^{2\theta}+3\bar{\eta})^2}{9(e^{2\theta}-3\bar{\eta})}}$$

- The results displayed in phase space show that there is a curve which attract most trajectories obtained for several initial conditions $-1.3 \le \theta_i \le -1 \quad 0.2 \le \dot{\theta}_i \le 0.5$ which provide that the inflationary trajectories are attractors.



$$\frac{\ddot{\theta}}{1-\dot{\theta}} + 3H\dot{\theta} + \frac{V_{,\theta}}{\theta} = 0$$



Constant-roll inflation in RSII holographic model

- The scenario in which the brane (with an effective tachyon field) is located at the boundary of the AdS_5 space is referred as the holographic braneworld.

- The effective four-dimensional Einstein equations on the holographic boundary of AdS5 yields a modified Friedmann equations

$$h^2 - \frac{1}{4}h^4 = \frac{\kappa^2}{3}\ell^4\rho \qquad \dot{h}\left(1-\frac{1}{2}h^2\right) = -\frac{\kappa^2}{2}\ell^3(p+\rho)$$

where h is a dimensionless Hubble expansion rate and the fundamental coupling is related to the AdS_5 curvature radius

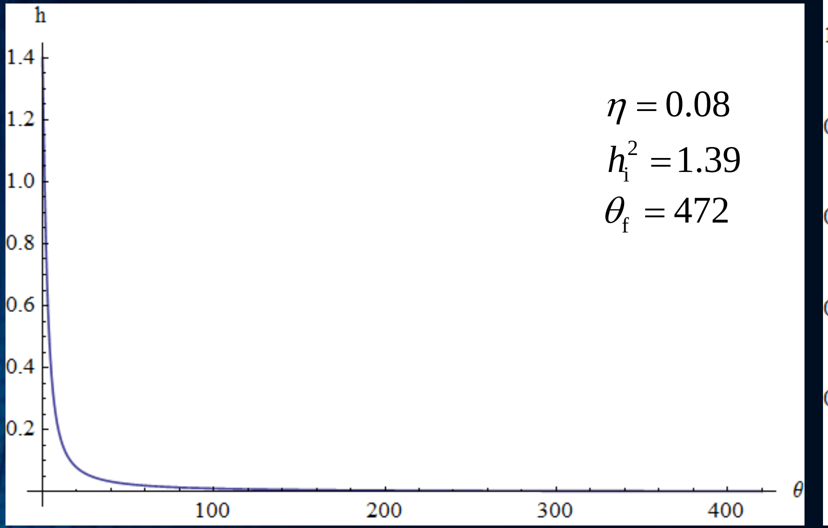
$$0 \le h^2 \le 2 \qquad \kappa^2 = \frac{8\pi G_{\mathrm{N}}}{\ell^2}$$

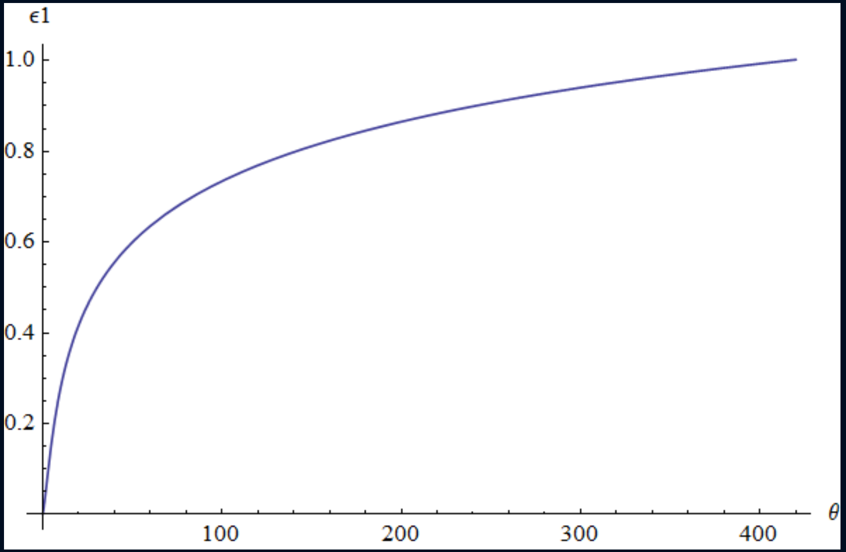
- From the general condition for constant-roll inflation using the Hamilton-Jacobi formalism one obtains

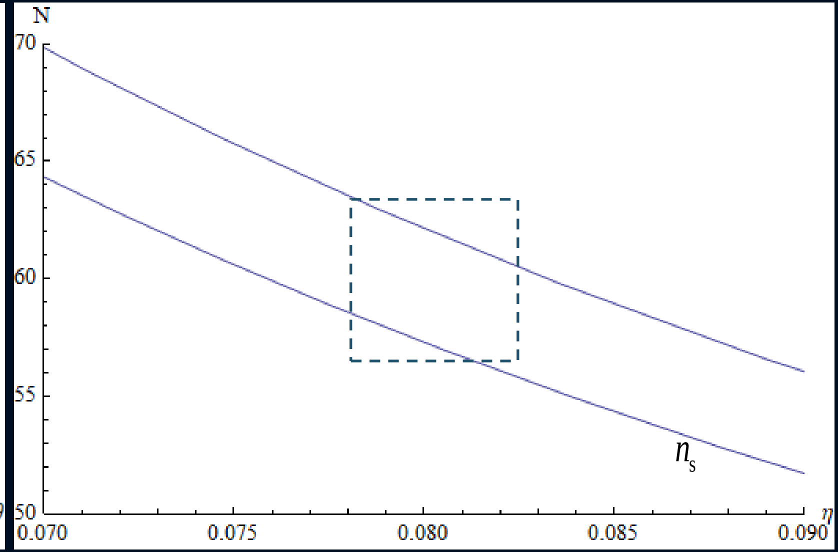
$$hh_{,\theta\theta} - 2h_{,\theta}^2\left(1+\frac{h^2}{4(1-\frac{1}{2}h^2)(1-\frac{1}{4}h^2)}\right) + \frac{3}{2\ell^2}\eta h^4\frac{1-\frac{1}{4}h^2}{1-\frac{1}{2}h^2} = 0 \qquad V = \frac{3}{\kappa^2}h^2(1-\frac{1}{4}h^2)\sqrt{1-\frac{4\ell^2}{9}\frac{h_{,\theta}^2}{h^4}\left(\frac{1-\frac{1}{2}h^2}{1-\frac{1}{4}h^2}\right)^2} \qquad \varepsilon_2 = 2\eta - \frac{h^2}{2(1-\frac{1}{2}h^2)(1-\frac{1}{4}h^2)}\varepsilon_1$$

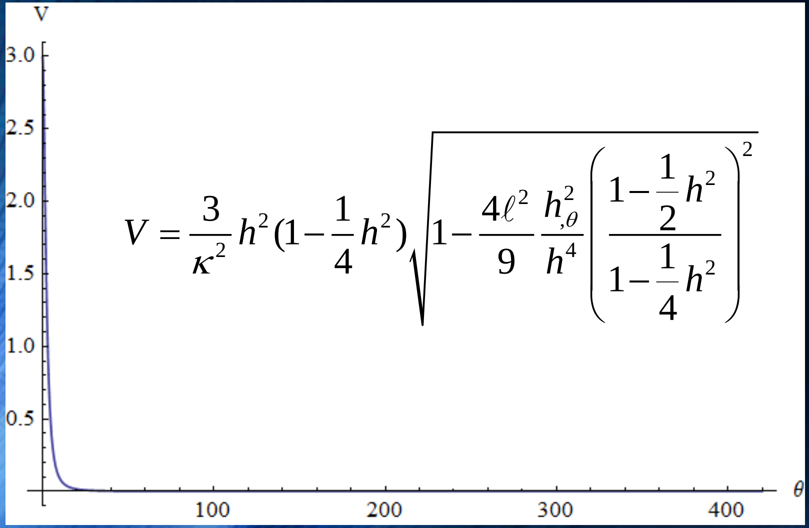
- The expressions obtained in the CRI in holography differ from those in CRI in the standard cosmology!

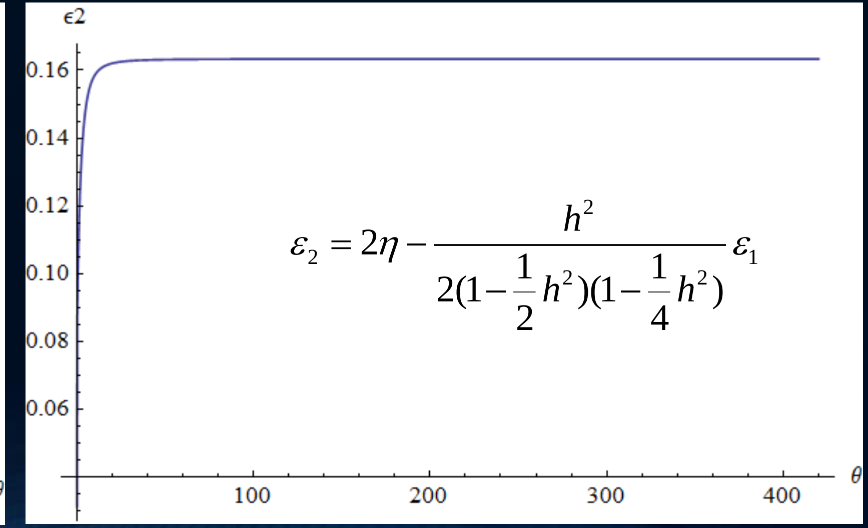
Constant-roll inflation in RSII holographic model

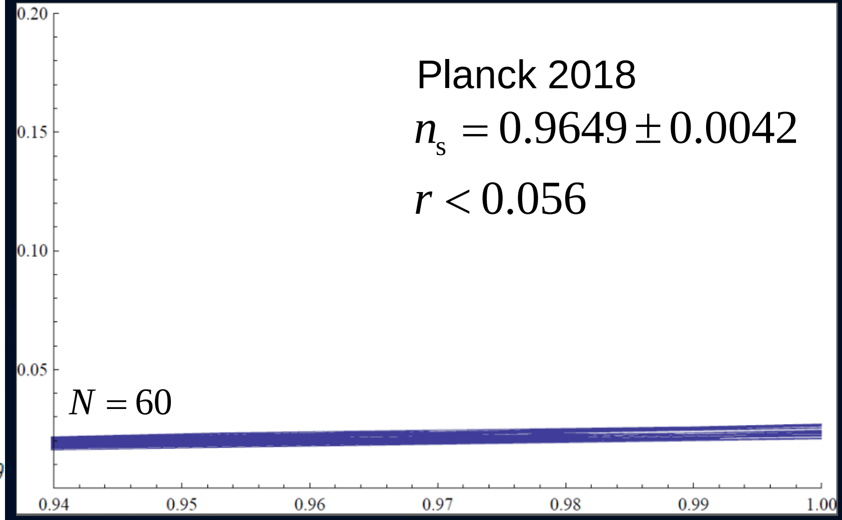












Conclusions

- We have studied the constant-roll inflation with tachyon field in RSII Cosmology, with constant slow-roll parameter η, and for fixed η.
- Its definition leads to differential equation for the Hubble expansion rate, which have the exact (4+1) solutions.
- We found Hubble slow-roll parameters (ε_1, ε_2) as a function of parameter η for all (4 nontrival) solutions H(θ).
- It was shown show that three of four solutions H(θ) provide a consistent inflationary model. Futhermore, and as very important, all solutions lead to the same function $\varepsilon_1(N)$ and $\varepsilon_2(N)$.
- We calculated the values of n_s and r and compared it with the latest Planck results.By comparing those values with constraints from observation data we estimate the parameter η. The better agreement is achieved for negative and small value of the parameter η.
- In addition, for standard and RSII cosmology we have calculated inflation parameters in the second order in the slow-roll parameters. No significant difference was obtained for the parameters in these two cases.
- A correct attractor behaviour was found.
- The model of CRI in holographic cosmology gives a lower value for number of e-fold and closer to typical value N=60 then the tachyon CRI in standard cosmology.

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THANK YOU FOR YOUR ATTENTION!