Invertibility as a necessary condition for Markovianity of the quantum dynamical maps

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THE BASIC CONCEPTS

$\rho(t) = \mathcal{G}(t,t_0)\rho(t_0)$ (0)

$\mathcal{G}(t,t_0)$ - dynamical map

divisibility

$\mathcal{G}(t,t_0) = V(t,s)\mathcal{G}(s,t_0),$ (1)

invertibility:

$\exists \mathcal{G}^{-1}(t,t_0) | \mathcal{G}(t,t_0)\, \mathcal{G}^{-1}(t,t_0) = \mathcal{I}$ (2)

time locality:

$\frac{d\rho(t)}{dt} = \mathcal{L}_t \rho(t)$ (3)

THE RESULT

- Equation (3) is necessary for quantum Markovianity whatsoever.

- There is a class $\mathcal{C}$ of the quantum dynamical maps for which:

Lemma. The three characteristics (1)-(3) are mutually equivalent for the $\mathcal{C}$ class of the quantum dynamical maps.

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Conclusion: Invertibility is a necessary condition for quantum Markovianity for the $\mathcal{C}$ class of the quantum dynamical maps – that can be routinely experimentally tested.

HISTORY

J. Jeknic-Dugic, M. Arsenijevic, M. Dugic, Problem Book in Open Quantum Systems Theory (in Serbian) -- aimed at the basic PhD-student education, but: *L. Lautenbacher et al, Phys. Rev. A* ***105****, 042421 (2022); P. Taranto, PhD Thesis, Quantum Information Processing: Thermodynamics, Complexity, and Multi-Time Phenomena, Vienna 2022; J. Joo, T. P. Spiller, New J. Phys.* ***25,*** *083041 (2023).*

Quantum (journal)

J. Phys. A

J. Math. Phys.

ABOUT PHYSICS

- The $\mathcal{C}$ class of the quantum dynamical maps is the "<u>only physics</u>" at the level of the <u>basic</u> physical laws (processes).

Definition. A *linear* and *completely positive* dynamical map Φ is in the so-called C class of dynamical maps if and only if the following requirements are simultaneously fulfilled: **(a)** the map is time continuous, in the sense it is defined on a continuous-time interval $t \in [t_\circ, t]$, **(b)** the map is a two-parameter map denoted $\Phi(t, t_\circ)$, $t \geq t_\circ$, **(c)** the map is smooth enough (ultraweak continuity), in the sense that, for positive a, $\lim_{a\downarrow 0} \Phi(t + a, t_\circ) = \Phi(t, t_\circ)$, for $t \geq t_\circ$, is well defined, **(d)** the map has the whole Banach space of statistical operators (density matrices) in its domain, and **(e)** the map is differentiable, i.e., that the (ultraweak) limit.

$d\Phi(t, t_\circ)/dt = \lim_{a\downarrow 0} [\Phi(t + a , t_\circ) - \Phi(t, t_\circ)]/a$

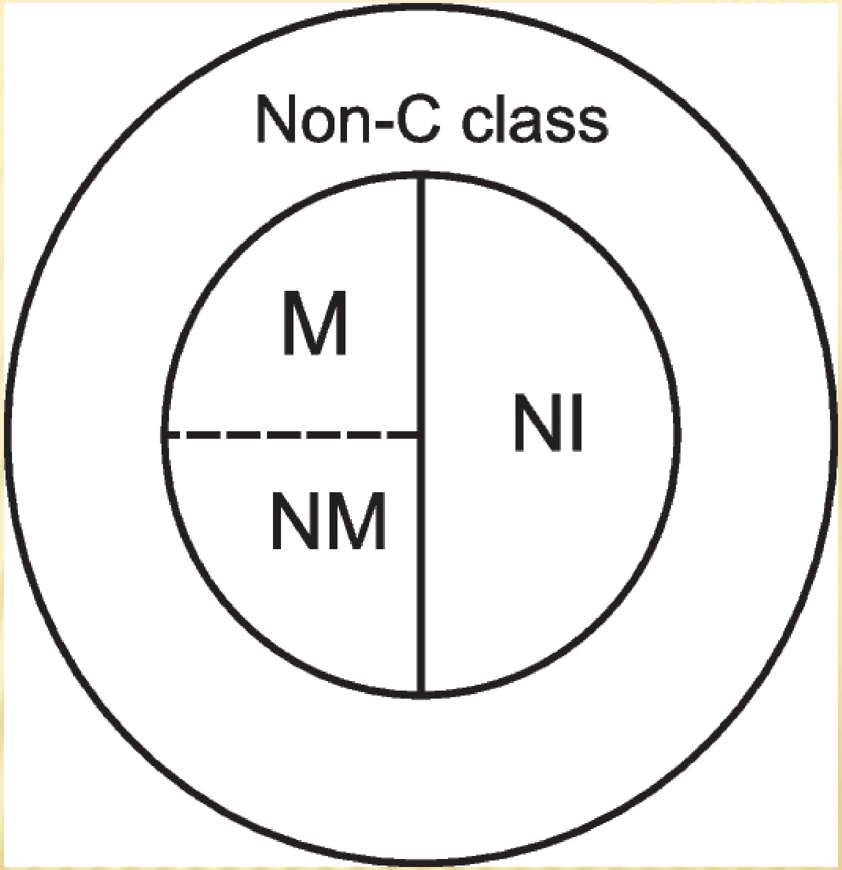
is well defined.

THE MERITS OF THE PAPER

Strategically: to reduce the ado in the field.

- There are not the simple and universal rules for Markovianity beyond invertibility (MIB), BUT there is a class of the maps for which the rules are very simple
- The BLP-Markovianity is useless for the C class processes
- Explains the procedures that can be found in the literature
- Sets a platform for the future search for MIB and the more general relations between (1)-(3) and Markovianity

FIGURE



Thank you for your attention!

DERIVATION OF THE AJL INEQUALITY

The relation (1) for the typical models reads:

$$\mathcal{G}(t,t_0) = \mathcal{G}(t,s)\mathcal{G}(s,t_0)$$

where $\mathcal{G}(t,t_0) = \mathcal{T}e^{\int_{t_0}^{t}\mathcal{L}_u du}$.

Presented in a matrix form, eq. (1) reads:

$g(t,t_0) = g(t,s)g(s,t_0)$. (*)

From (*) it follows $detg(t,t_0) = detg(t,t_0)detg^{-1}(s,t_0)detg(s,t_0)$.

On the use of the time-splitting formula and det $e^A = e^{trA}$ follows:

$detg(t,t_0) = detg(s,t_0)e^{\int_s^t tr\mathcal{L}_u du}$, (AJL)

which is the AJL inequality (often given for t_0=0).

The inverse [(AJL) implies (1)] is, in general, not correct. In general, neither $\mathcal{G}(t,s)\mathcal{G}(s,t_0) = \mathcal{G}(s,t_0)\mathcal{G}(t,s)$ nor $\mathcal{G}(t,s) = \mathcal{T}e^{\int_s^t \mathcal{L}_u du}$ is fulfilled.